C3 June 07

1) $\ln a + \ln b = \ln ab$

$\ln x + \ln 3 = \ln 3x = \ln 6 \Rightarrow 3x = 6 \Rightarrow \underline{x = 2}$

b) $e^x + 3e^{-x} = 4$

$(\times e^x) \quad (e^x)^2 + 3 = 4(e^x) \Rightarrow (e^x)^2 - 4(e^x) + 3 = 0$

let $y = e^x \Rightarrow y^2 - 4y + 3 = 0 \Rightarrow (y-3)(y-1) = 0 \Rightarrow y = 3, y = 1$

$\Rightarrow e^x = 3 \Rightarrow \ln e^x = \ln 3 \Rightarrow \underline{x = \ln 3}$

$\Rightarrow e^x = 1 \Rightarrow \underline{x = 0}$

2) $\frac{2x+3}{x+2} - \frac{9+2x}{(2x-1)(x+2)} \Rightarrow \frac{(2x+3)(2x-1)}{(x+2)(2x-1)} - \frac{9+2x}{(x+2)(2x-1)}$

$\Rightarrow \frac{4x^2+4x-3-9-2x}{(x+2)(2x-1)} \Rightarrow \frac{4x^2+2x-12}{(x+2)(2x-1)} = \frac{(x+2)(4x-6)}{(x+2)(2x-1)}$

$\Rightarrow f(x) = \underline{\frac{4x-6}{2x-1}}$

b) $u = 4x-6 \quad v = 2x-1 \quad f'(x) = \frac{vu' - uv'}{v^2} = \frac{8x-4-(8x-12)}{(2x-1)^2}$

$u' = 4 \quad v' = 2$

$f'(x) = \underline{\frac{8}{(2x-1)^2}}$

3) $y = x^2 e^x \quad u = x^2 \quad v = e^x \quad y' = vu' + uv'$

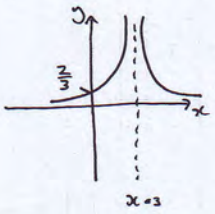
$u' = 2x \quad v' = e^x \quad y' = 2xe^x + x^2 e^x$

$y' = x(x+2)e^x$

b) TP when $y' = 0$ hence $x = 0, x = -2, e^x = 0 \Rightarrow$ Impossible.

$x = 0 \Rightarrow y = 0 \quad \underline{(0,0)} \qquad x = -2 \Rightarrow y = 4e^{-2} \quad \underline{(-2, 4e^{-2})}$

5c) $|g(x)|$



asymptotes x axis, $x = 3$

d) $\left|\frac{2}{x-3}\right| = 3$ crosses both parts of graph

$\frac{2}{x-3} = 3 \Rightarrow 2 = 3x - 9 \Rightarrow 3x = 11 \Rightarrow \underline{x = \frac{11}{3}}$

$-\left(\frac{2}{x-3}\right) = 3 \Rightarrow -2 = 3x - 9 \Rightarrow 3x = 7 \Rightarrow \underline{x = \frac{7}{3}}$

6) $R\sin(\theta + \alpha) = a\sin\theta + b\cos\theta$

$3\sin x + 2\cos x$

$b = 2 = \sin\alpha \quad \tan\alpha = \frac{2}{3}$

$a = 3 = \cos\alpha \quad \alpha = 0.588$

$R = \sqrt{2^2 + 3^2} = \sqrt{13}$

$\underline{\sqrt{13}\sin(x + 0.588)}$

b) max value of $\sin(\ ) = 1$

max value of $\sqrt{13} \times \sin(\ ) = \sqrt{13}$

max value of $(3\sin x + 2\cos x)^4 = (\sqrt{13})^4 = 13^2 = \underline{169}$

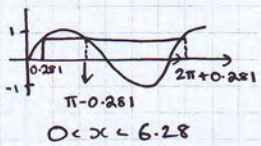
c) $\sqrt{13}\sin(x + 0.588) = 1 \Rightarrow \sin(x + 0.588) = \frac{1}{\sqrt{13}}$

$\Rightarrow x + 0.588 = \sin^{-1}\left(\frac{1}{\sqrt{13}}\right) = 0.281$

$x + 0.588 = 0.281 \Rightarrow x = -0.307$ ✗

$x + 0.588 = \pi - 0.281 \Rightarrow x = 2.273$ ✓

$x + 0.588 = 2\pi + 0.281 \Rightarrow x = 5.976$ ✓



$0 < x < 6.28$

3c) $y' = (x^2+2x)e^x \quad u = x^2+2x \quad v = e^x$

$u' = 2x+2 \quad v' = e^x$

$y'' = (2x+2)e^x + (x^2+2x)e^x$

$y'' = \underline{(x^2+4x+2)e^x}$

d) at $(0,0) \quad y'' = 2e^0 = 2$ +ve ∪ minima

at $(-2, 4e^{-2}) \quad y'' = (4-8+2)e^{-2} = -2e^{-2}$ -ve ∩ maxima

4) $f(x) = -x^3 + 3x^2 - 1 = 0 \Rightarrow -x^3 + 3x^2 = 1$

$\Rightarrow 3x^2 - x^3 = 1 \Rightarrow x^2(3-x) = 1 \Rightarrow x^2 = \frac{1}{3-x} \Rightarrow x = \sqrt{\frac{1}{3-x}}$

$\underline{x_1 = 0.6}, \underline{x_2 = 0.6455}, \underline{x_3 = 0.6517}, \underline{x_4 = 0.6526}$

c) Show $x = 0.653$ is a root test 0.6525 and 0.6535.

$f(0.6525) = -0.00054$ -ve $\quad f(0.6535) = 0.002$ +ve

from change of sign rule 0.653 (3dp) is a root.

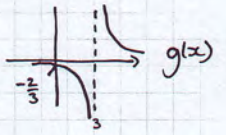
5) $f(g(4)) = f\left(\frac{2}{4-3}\right) = f(2) = \ln(4-1) = \underline{\ln 3}$

b) $y = \ln(2x-1) \Rightarrow x = \ln(2y-1) \Rightarrow e^x = 2y - 1 \Rightarrow y = \underline{\frac{e^x + 1}{2}}$

domain $x \in \mathbb{R}$.

c) 

$y = \frac{2}{x} = f(x) \qquad y = \frac{2}{x-3} \quad f(x-3) \quad 3 \rightarrow$



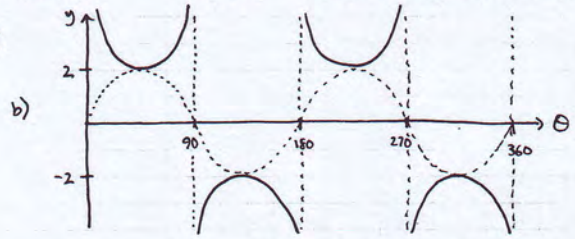
when $x = 0, \frac{2}{x-3} = \frac{2}{-3} = -\frac{2}{3}$

horizontal asymptote = x axis vertical asymptote $\Rightarrow \underline{x = 3}$

7) $\frac{\sin\theta}{\cos\theta} \times \frac{\sin\theta}{\sin\theta} + \frac{\cos\theta}{\sin\theta} \times \frac{\cos\theta}{\cos\theta} \Rightarrow \frac{\sin^2\theta + \cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta} = \frac{2}{2\sin\theta\cos\theta} = \frac{2}{\sin 2\theta}$

$= \underline{2\operatorname{Cosec} 2\theta}$

b)

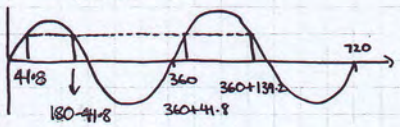


c) $2\operatorname{Cosec} 2\theta = 3 \Rightarrow \operatorname{Cosec} 2\theta = \frac{3}{2} \Rightarrow \frac{1}{\sin 2\theta} = \frac{3}{2} \Rightarrow \sin 2\theta = \frac{2}{3}$

$\Rightarrow 2\theta = \sin^{-1}\left(\frac{2}{3}\right) = 41.8°$

$2\theta = 41.8, 139.2, 401.8, 499.2$

$\theta = \underline{20.9°}, \underline{69.1°}, \underline{200.9°}, \underline{249.1°}$



8) $x = De^{-\frac{1}{8}t} \quad$ a) $D = 10, t = 5 \quad x = 10e^{-\frac{5}{8}} = \underline{5.353}$ mg

b) $D = 15.353, t = 1$ hr $\quad x = 15.353e^{-\frac{1}{8}} = \underline{13.549}$ m

c) $3 = 15.353e^{-\frac{1}{8}t} \Rightarrow e^{-\frac{1}{8}t} = 0.195 \Rightarrow -\frac{1}{8}t = \ln 0.195$

$-\frac{1}{8}t = -1.633 \Rightarrow t = 13.06$ hrs $\quad \underline{13}$hrs $\underline{3}$min $\underline{42}$sec